WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

   (a) Evaluate $\lim\limits_{x\to0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

   (b) If (2, 5/2) is known to be a point of inflection of the curve $3x^2y+\alpha x+\beta y=0$, then find the value of α and β.

   (c) Find the interval where the curve $y=e^x(\cos x+\sin x)$ is concave upwards or downwards for $0<x<2\pi$.

   (d) Write the equation $4xy=1$ in terms of a rotated rectangular $x'y'$-system if the axes are turned through an angle $\tan^{-1}2$.

   (e) Show that the abscissa of the points of inflexion on the curve $y^2=f(x)$ satisfy the equation $\{f'(x)\}^2=2f(x)f''(x)$.

   (f) Find the equation of the generating lines of the hyperboloid $yz+2zx+3xy+6=0$ which pass through the point $(-1, 0, 3)$.

   (g) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2=4$, $z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3 units.

   (h) Find the value of a and b for which the differential equation $(3a^2x^2+by\cos x)\,dx+(2\sin x-4ay^3)\,dy=0$ is exact.

   (i) Show that the equation $\frac{dy}{dx}=2y^{1/2}$, $y(0)=0$ has no unique solution.

2. (a) If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x$, prove that 4

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

   (b) Prove that the points of inflexion of the curve $y^2(x-a)=x^2(x+a)$ subtend an angle $\frac{\pi}{3}$ at the origin. 4

3. (a) Show that envelope of the lines drawn at right angles to the radii vectors of the cardioid $r=a(1+\cos\theta)$ through their extremities is given by $r=2a\cos\theta$. 4

   (b) Find the asymptotes of the curve $r=\frac{a}{\frac{1}{2}-\cos\theta}$. 4

4. (a) Trace the curve given by the equation 4

$$y^2=x^2\left(\frac{a+x}{a-x}\right).$$

(b) Show that the equation $4x^2-4xy+y^2+2x-26y+9=0$ represents a parabola whose latus rectum is $2\sqrt{5}$ units. 4

5. (a) Prove that length of the arc of the parabola $y^2=4ax$ which is intercepted between the points of intersection of the parabola and the straight line $3y=8x$ is $a\left(\log 2+\frac{15}{16}\right)$. 4

(b) A sphere of constant radius 'd' through the origin and intersects the co-ordinate axes in P, Q, R. Prove that the centroid of the triangle PQR lies on the sphere $9(x^2+y^2+z^2)=4d^2$. 4

6. (a) Find the equation of the sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point $(2,-4,6)$. 4

(b) Find the equation of the cylinder whose generators are parallel to the straight line $2x=y=3z$ and which passes through the circle $x^2+z^2=6,\ y=0$. 4

7. (a) Through a variable generator $x-y=\lambda,\ x+y=\frac{2z}{\lambda}$ of the paraboloid $x^2-y^2=2z$ a plane is drawn, making an angle $\frac{\pi}{4}$ with the plane $x=y$. Find the locus of the point at which it touches the paraboloid. 4

(b) The curve that an idealised hanging chain or cable assumes when supported at its ends and acted on solely by its own weight is called a catenary. The equation of this curve is 4

$$y=a\cosh\left(\frac{x}{a}\right)=\frac{a}{2}(e^{x/a}+e^{-x/a})$$

Find the arc length of the curve between the points where it is cut by $y=2a$.

8. (a) Determine the surface area of the solid obtained by rotating $y=\sqrt{9-x^2}$, $|x|\le 2$ about the x-axis. 4

(b) Show that the following first order ode is exact and hence solve it. 4

$$\left(\frac{1+8xy^{2/3}}{x^{2/3}y^{1/3}}\right)dx+\left(\frac{2x^{4/3}y^{2/3}-x^{1/3}}{y^{4/3}}\right)dy=0.$$

9. (a) Find suitable integrating factor of the following ode and hence solve it. 4

$$(6+12x^2y^2)+\left(7x^3y+\frac{x}{y}\right)\frac{dy}{dx}=0.$$

(b) Find singular solution of $9\left(\frac{dy}{dx}\right)^2(2-y)^2=4(3-y)$. 2

(c) Solve: $(4x^2y-6)\,dx+x^3dy=0$. 2

10.(a) Determine the constants a, b, c such that 4

$$\lim_{x\to 0}\frac{x(a+b\cos x)+c\sin x}{x^5}=\frac{1}{60}$$

(b) Show that the differential equation of the circles through the intersection of the circle $x^2+y^2=1$ and the line $x-y=0$ is given by 4

$$(x^2-2xy-y^2+1)\,dx+(x^2+2xy-y^2-1)\,dy=0.$$

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